Conjugate Fuzzy Subgroup

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ABSTRACT

We introduce here the notion of (i) a conjugate fuzzy subgroup (ii) a fuzzy middle coset.In this paper we give necessary and sufficient condition for a conjugate fuzzy subgroup of a fuzzy group.The aim of the paper is to invastigate conjugate fuzzy subgroup of a group from a general point of view.

Keywords - Fuzzy subgroup, fuzzy middle coset, conjugate fuzzy subgroup.

1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh[2]. Rosenfeld [1] applied this concept to the theory of groupoids and groups. In [4] the notion of a conjugate fuzzy subgroup of a fuzzy group was introduced and studied. The purpose of this paper is to continue the study of conjugate fuzzy subgroup of a fuzzy group . In this paper we investigate further the theory of fuzzy group and obtain analogs of a number of basic results of conjugate fuzzy subgroup and fuzzy middle coset.

A. Some Definition

Definition 1: Let X be a non empty set.A fuzzy subset S of the set X is a function $S : X \rightarrow [0,1]$.

Definition 2: Let G be a group.A fuzzy subset S of a group G is called a fuzzy subgroup of the group G if

(i) $S(x,y) \geq min\,\{S(x), S(y)\}\ \forall\, x,y\ \in G.$
(ii) $S(x^{-1}) = S(x)\quad \forall\ x\ \in G.$

Definition 3: Let S be a fuzzy subset of a set X.For $t \in [0,1]$, the set $S_t = \{x \in X \mid S(x) \geq t\}$ is called a level subset of the fuzzy subset S.

Definition 4: Let G be a group. A fuzzy subgroup S of G is called normal if $S(x) = S(y^{-1}xy)\quad \forall\, x,y\ \in G.$

A. Conjugate Fuzzy Subgroup

Definition A1: Let μ and S be two fuzzy subgroup of G then μ and S are said to be conjugate fuzzy subgroup of G if for some $g \in$ G , $\mu(x) = S(g^{-1}xg)$ for every $x \in$ G.

Theorem A2: Let μ and λ are conjugate fuzzy subgroup of the group G then $o(\mu) = o(\lambda)$.

Proof: Refer [4] for proof.

Theorem A3: Let μ and λ be any two fuzzy subgroup of the group G. Then
μ and λ are conjugate fuzzy subgroup of G iff μ=λ

Proof: Let μ and λ are conjugate fuzzy subgroup of the group G. To show that μ=λ. Since μ and λ are conjugate fuzzy subgroup therefore $\exists\ g \in$ G such that $\mu(x) = \lambda(g^{-1}xg)\ \forall\, x \in$ G. Let $x = gx \in$ G, then $\mu(gx) = \lambda(g^{-1}gxg) \Rightarrow \mu(gx) = \lambda(xg)$. For some $g = e \in$ G, we have $\mu(ex) = \lambda(xe) \Rightarrow \mu(x) = \lambda(x) \Rightarrow \mu = \lambda$. Convers, to show that μ and λ are conjugate fuzzy subgroup. Let $\mu = \lambda \Rightarrow \mu(x) = \lambda(x) \Rightarrow \mu(x) = \lambda(e^{-1}xe)\ \forall\, x, e \in$ G. Hence μ and λ are conjugate fuzzy subgroup of the group G.

Theorem A4: Let λ be a fuzzy subgroup of the group G and μ be a fuzzy subset of G. If λ and μ are conjugate fuzzy subgroup of the group G then μ is a fuzzy subgroup of the group G.

Proof: Let G be a group with identity e.
If λ and μ are conjugate fuzzy subgroup of the group G therefore $\exists$ g $\in$ G such that $\lambda(x) = \mu\,(g^{-1}xg)\ \forall$ x $\in$ G. Also $\mu(x) = \mu(exe) = \mu(g^{-1}gxg^{-1}g) = \lambda(gxg^{-1})$.
To show that μ is a fuzzy subgroup of the group G.
Since λ is a fuzzy subgroup of the group G therefore $\lambda(x,y) \geq \min\{\lambda(x), \lambda(y)\}\ \forall$ x, y $\in$ G and $\lambda(x^{-1}) = \lambda(x)\ \forall$ x $\in$ G. Now $\mu(xy) = \mu(exeye) = \mu(g^{-1}gxg^{-1}gyg^{-1}g) = \lambda(gxg^{-1}gyg^{-1}) \geq \min\{\lambda(gxg^{-1}), \lambda(gyg^{-1})\} \geq \min\{\mu(x), \mu(y)\}$. Also $\mu(x^{-1}) = \lambda(gx^{-1}g^{-1}) = \lambda(gxg^{-1}) = \mu(x)$. Hence μ is a fuzzy subgroup of the group G.

B. Fuzzy Middle Coset

Definition B1: Let S be a fuzzy subgroup of group G then for any $a, b \in$ G, a fuzzy middle coset aSb of the G is defined by $(aSb)(x) = S(a^{-1}x\, b^{-1})$ $\forall\, x \in$ G.

Theorem B2: If S is a fuzzy subgroup G then for any $a \in$G, the fuzzy middle coset aSa^{-1} of the group of the group G is also a fuzzy subgroup of the group G.

Proof : Let G be a group and S: G→ [0,1] be a fuzzy subgroup. For any $a \in$G, aSa^{-1} is a fuzzy middle coset of G i.e. $(aSa^{-1})(x) = S(a^{-1}xa)$ $\forall\, x \in$ G. To show that aSa^{-1} is a fuzzy subgroup of G.Let S is a fuzzy subgroup,
therefore $S(x^{-1}) = S(x)$. Now$(aSa^{-1})(x^{-1}) = S(a^{-1}x^{-1}a) = S((a^{-1}x^{-1}a)^{-1}) = S(a^{-1}xa) = (aSa^{-1})(x)$ Also, $aSa^{-1}(xy) = S(a^{-1}xya) = S(a^{-1}xaa^{-1}ya) \geq min[S(a^{-1}xa), S(a^{-1}ya)] \geq min[aSa^{-1}(x), aSa^{-1}(y)]$. Hence aSa^{-1} is a fuzzy subgroup of G.

Theorem B3: Let S be any fuzzy subgroup of group G and aSa^{-1} be a fuzzy middle coset of G then $o(aSa^{-1})$ = $o(S)$ for any $a \in$G.

Proof: By the theorem 3.2- aSa^{-1} is a fuzzy subgroup of G. Thus $(aSa^{-1})(x) = S(a^{-1}xa)$ $\forall\, x \in$ G. Therefore S and aSa^{-1} are conjugate fuzzy subgroup of G. By the theorem A.2- $o(aSa^{-1})$ = $o(S)$ for any $a \in$G.

CONCLUSIONS

The study of fuzzy set theory has become increasingly important in the wake of fast technological development and increasing complexities in real world decision making problems. The fuzzy set theory technique are now considered as an effective and powerful aid towards solving problems of management decision making, computer science, medical science, artificial intelligence etc. So we define in this paper algebraic data about fuzzy.

ACKNOWLEDGEMENT

The author is thankful to this work is dedicated to my parents and my husband and I also very grateful to my guide Dr. U. K. Shrivastava, Department of Mathematics, Govt. Bilasa Girls P. G. Collage, Bilaspur(C.G.) without their help couldn't complete my work.

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